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MISCELLANEA

USE OF MATHEMATICAL MODELS OF THE IONOSPHERE FOR STUDYING THE PROPAGATION OF ELECTROMAGNETIC WAVES

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Using theoretical and experimental models, the results of a computational experiment on the propagation of radiowaves are presented. It is shown that to more accurately calculate the angle of arrival, it is necessary to take into account the lower ionosphere, i.e., the D region.

Introduction. In order to carry out a reliable determination of the point of arrival of an electromagnetic signal sent to the atmosphere under terrestrial conditions, one has to know the parameters of the ionospheric plasma. Since their experimental determination is difficult, the principal method for determining them could be physicomathematical models allowing one to calculate altitude-time distributions of the ionospheric parameters for various heliogeophysical conditions. The adequacy of the results obtained was proved by comparing them with experimental data. In this work we present the results of calculations with the joint use of the models of the ionosphere and of the propagation of radio paths.

Model of the Propagation of Radio Waves. Based on the geometric optics approximation [1], the wave equation that describes the interaction of an electromagnetic wave with the ionosphere is reduced to a system of ordinary differential equations:

$$\frac{d\mathbf{r}}{d\tau} = \frac{\partial \Gamma}{\partial \mathbf{p}}\,, \quad \frac{d\mathbf{p}}{d\tau} = -\frac{\partial \Gamma}{\partial \mathbf{r}}\,, \quad \frac{d\Psi}{d\tau} = \mathbf{p}\,\frac{\partial \Gamma}{\partial \mathbf{p}}\,, \quad \mathbf{p} = \nabla \Psi \tag{1}$$

at the initial conditions

$$\mathbf{r}(0) = \mathbf{r}_0(\alpha, \beta)\,, \quad \mathbf{p}(0) = \mathbf{p}_0(\alpha, \beta)\,, \quad \Psi(0) = \Psi_0\,.$$

The system is solved numerically by the method of characteristics that allows one to apply standard numerical methods (the Runge–Kutta method). After the beam trajectory in the ionosphere has been determined, it is possible to calculate various characteristics of the field at the reception point, each of which is obtained by integration of the corresponding parameter along the trajectory. Thus, for the field amplitude we obtain

$$E = E_0(\alpha, \beta)\left[q\,\frac{J(\tau, \alpha, \beta)}{J(0, \alpha, \beta)}\right]^{-1/2}, \qquad q = \frac{|\mathbf{a}|}{|\mathbf{v}^2\mathbf{p}|}\,\frac{\eta^2 \cos\zeta}{n^2 + \sin\zeta}\,.$$

Expressions for the phase and group paths as well as for integral absorption have the following forms:

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$$P = \int_S \mu \cos \zeta ds\,, \quad P' = \int_S \mu' \cos \zeta ds\,, \quad L = \int_S \chi \cos \zeta ds\,, \tag{2}$$

where $\mu = \mathrm{Re}\,(n(\mathbf{r}))$; $\mu' = \mathrm{Re}\,(n'(\mathbf{r}))$.

For short-wave (SW) radio signals in an isotropic case, with the magnetic field of the Earth being neglected, we will represent the system of equations (1) as

$$\frac{d\mathbf{r}}{d\,(ct)} = \frac{c}{\omega}\mathbf{a}\,, \quad \frac{d\mathbf{a}}{d\,(ct)} = \frac{2\pi e^2}{m\omega^2 c}\nabla N_{\mathrm{e}}\,, \quad \frac{d\Psi}{d\,(ct)} = \frac{\omega}{c}\left(1 - \frac{4\pi e^2}{m\omega^2 N_{\mathrm{e}}}\right). \tag{3}$$

For a numerical solution of system (3) that determines the trajectory of the beam, we must select an optimal interpolation for N_{e} and its gradients in r, θ, and φ, i. e., N_r, N_θ, and N_φ. Mathematical models serve as methods of assigning such values.

Work [2] contains the basic requirements imposed on theoretical models of the ionosphere: the characteristic dimensions of the ionosphere inhomogeneities over the altitude l_h, longitude l_θ, and latitude l_φ must be higher than the corresponding internodal distances over the altitude, longitude, and latitude, i.e., it is necessary that the equalities $l_h \geq \Delta h$, $l_\theta \geq \Delta\theta$, and $l_\varphi \geq \Delta\varphi$ be fulfilled and that the integration step $s \leq l_h$, l_θ, l_φ. The condition of geometric approximation is also valid for a small change in the gradient of electron concentration at a distance of the order of the wavelength.

Models of the Ionosphere. The proposed model allows one to calculate the space-time distribution of the following components: N_2, O, O_2, O_3, $O(^1D)$, $O(^1S)$, $O_2(^1\Delta_g)$, H, H_2, OH, H_2O, H_2O_2, $N(^4S)$, $N(^2D)$, NO, NO_2, CO, CO_2, O^+, O_2^+, NO^+, O_4^+, N^+, N_2^+, N_{e}, Y^+, and Y^-.

The continuity equation for ions is written in the form

$$\frac{\partial n_j}{\partial t} = \frac{\partial}{\partial z}\left(D_j \frac{\partial n_j}{\partial z} + \overline{D}_j n_j\right) - \alpha_j n_j + F_j\,, \tag{4}$$

where

$$D_j = kT\left(\sum_{j\neq k} \mu_{jk}\nu_{jk}\sin^2 I\right)^{-1}; \quad \overline{D}_j = \frac{\sum_{k\neq j} \mu_{jk}\nu_{jk}V_{kz}\sin^2 I}{\sum_{k\neq j}\mu_{jk}\nu_{jk}} + D_j\left(\frac{1}{N_{\mathrm{e}}}\frac{\partial N_{\mathrm{e}}}{\partial z} + \frac{2}{T}\frac{\partial T}{\partial z} + \frac{1}{H_j}\right),$$

and for neutral components

$$\frac{\partial n_k}{\partial t} = \frac{\partial}{\partial z}\left(D_k \frac{\partial n_k}{\partial z} + \overline{D}_k n_k\right) - \alpha_k n_k + F_k\,, \tag{5}$$

where

$$D_k = D_{k\mathrm{m}} + D_{\mathrm{t}}\,; \quad D_{k\mathrm{m}} = kT\left(m_k \sum_{k\neq j} S_{kj} n_j\right)^{-1};$$

$$\overline{D}_k = \frac{D_{k\mathrm{m}}}{H_k} + \frac{D_{\mathrm{t}}}{H} + \frac{(D_{k\mathrm{m}} + D_{\mathrm{t}})}{T}\frac{\partial T}{\partial z} - V_k\,; \qquad V_k = \frac{\sum_{k\neq j} S_{kj} n_j V_{j\mathrm{m}}}{\sum_{k\neq j} n_j S_{kj}}\,;$$

D_t was given empirically depending on the season.

The total concentration of positive and negative ion-links is calculated from the equation

$$\frac{dY^{\pm}}{dt} = P^{\pm} - \alpha^{\pm} Y^{\pm}, \tag{6}$$

where the rate of formation of the positive ion-links and the loss coefficient will be presented as

$$P^{+} = B_{NO^+}[NO^{+}] + B_{O_2^+}[O_2^{+}], \quad \alpha^{+} = \alpha_{Y^+} N_e + \alpha^{*} Y^{-}, \quad \alpha_{Y^+} = 10^{-5}\ \text{cm}^{-3}\cdot\text{sec}^{-1}, \quad \alpha^{*} = 10^{-7}\ \text{cm}^{-3}\cdot\text{sec}^{-1}.$$

For negative ions-links

$$P^{-} = \bar{x}\,[O_2^{-}], \quad \alpha^{-} = Y^{+}\cdot 10^{-7} + \gamma.$$

The electron concentrations was calculated from the equation

$$\frac{dN_e}{dt} = P_e - \alpha_{ef} N_e \tag{7}$$

with the fulfillment of the quasi-neutrality condition being controlled: $N_e + Y^{-} = Y^{+} + \sum_i N_i$, where $\sum_i N_i$ is the total concentration of positive ions. The parameters B_{NO^+}, $B_{O_2^+}$, $\bar{x}$, α_{ef}, and γ are calculated following the selected photochemical schemes [3].

In order to calculate the altitude-time distribution of the concentration of O^+ and H^+ along the magnetic force line of the Earth, the following system of equations was used:

the continuity equation

$$\frac{\partial n_i}{\partial t} + B\frac{\partial}{\partial s}\left(\frac{1}{B} n_i V_i\right) = F_i - \alpha_i n_i, \tag{8}$$

the motion equation

$$n_i m_i \left(\frac{\partial V_i}{\partial t} + V_i \frac{\partial V_i}{\partial s}\right) + \frac{\partial P_i}{\partial s} = - n_i m_i g \sin I + n_i \sum_j S_{ij}(V_j - V_i) + n_i R_{in}(V_{nx}\cos I - V_i) - \frac{n_i}{N_e}\frac{\partial P_e}{\partial s}, \tag{9}$$

the energy equation

$$\frac{3}{2} k N_e \frac{\partial T_e}{\partial s} = B\frac{\partial}{\partial s}\left(\frac{1}{B}\lambda_e \frac{\partial T_e}{\partial s}\right) + \sum_i \frac{3 m_e N_e}{m_i}\nu_{ei} k (T_i - T_e) + Q_e - L_{e,n}, \tag{10}$$

$$\frac{3}{2} k N_i \frac{\partial T_i}{\partial t} = B\frac{\partial}{\partial s}\left(\frac{1}{B}\lambda_i \frac{\partial T_i}{\partial s}\right) + 3 n_i \nu_{ie} k (T_e - T_i) + \sum_n \frac{3 m_i n_i}{m_i + m_n}\nu_{in} k (T_n - T_i) + Q_i - L_i. \tag{11}$$

The system was described in detail in [4, 5]. The concentrations of neutral components, their temperatures, and pressure gradients were calculated by the model of [6].

The calculations of the above-enumerated parameters were performed for calm geophysical conditions, mean solar activity, and middle latitudes (45^{o} n. l.).

Results of Computational Experiment. The results of calculations of the altitude-time distribution of N_e for N_mF2 and different altitudes are presented in Fig. 1.

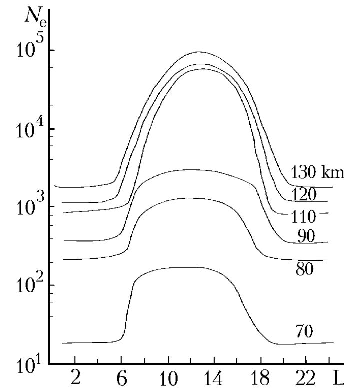

Fig. 1. Distribution of N_e calculated by the model of [3]. N_e, m^{-3}.

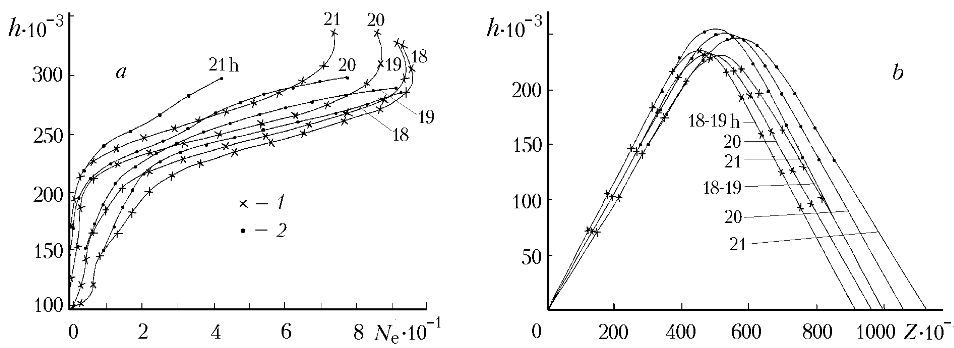

Fig. 2. Profiles of electron concentration (a) and of the SW radiopath (b) for different time moments (LT): 1) model of [8]; 2) experiment in [7].

To check the reliability of the prediction of the propagation of SW radio signals it is necessary to predict the state of the ionosphere by a dynamic model, and to calculate their paths; the obtained results on calculation of the paths are to be compared with experimental data. The reliability of the prediction of the ionosphere state can be verified with the aid of ionograms.

The above-indicated checking was made preliminarily using the experimental data on incoherent scattering for 09.06.1969. According to [7], the graphs of the electron concentration over the Sverdlovsk–Kaliningrad path were constructed with a step in the altitude of 7.5°, which corresponds to a time step of 0.5 h. Over the altitude the profiles were approximated by a spline with a nonuniform grid. In Fig. 2a these profiles are represented by curves with dots, where the first profile corresponds to 18 h and is located near Kaliningrad, the last one — to 21 h, over Sverdlovsk. Also presented here are the profiles (1) calculated according to the dynamic model of [8]. The profiles were obtained for the same conditions as in [3]. It is seen from Fig. 2a that the difference between the experimental and predicted profiles is equal to about 20%.

Figure 2b presents the results of calculation of paths for incoherent scattering and the dynamic model of the ionosphere, respectively. The paths were calculated for the frequency $f = 10$ MHz and exit angles of 25°, 27°, and 29°. The results for the same angles differ in the range by 10%. As was expected, the paths for the incoherent scattering pass higher, since here the ionosphere begins from the 150th km. Thus, with the error in the simulation of the ionosphere being 20%, the error of calculation of the routs comes to 10%.

Conclusions. It is shown that the obtained results on the physicomathematical models of the ionospheric plasma of the Earth can be used successfully in the problems of the propagation of electromagnetic waves, and that knowledge of the parameters D and E of the ionospheric regions is needed for a more accurate determination of the angle of arrival.

NOTATION

a, wave vector; B, magnetic induction, Tl; c, speed of light in a vacuum, m·sec^{-1}; D, diffusion coefficient, m^2·sec^{-1}; E, amplitude of the field; e, velocity of electrons, m·sec^{-1}; F, rate of the formation of particles, m^{-3}·sec^{-1}; g, free fall acceleration, m·sec^{-2}; H, scale of heights, m; h, step of integration over the height, m; I, magnetic inclination, deg; J, Jacobian; k, Boltzmann constant, J·K^{-1}; L, coefficient of gas cooling, W·sec^{-1}; LT, local time, h; m, mass, kg; $n(\mathbf{r})$, rarefaction index; $n'(\mathbf{r})$, group rarefaction index; N_e, concentration of electrons, m^{-3}; n_i, concentration of ions, m^{-3}; N_mF2, concentration of the maximum of the F2 layer of the Earth's ionosphere, m^{-3}; P, gas pressure, Pa; **p**, generalized pulse; Q, coefficient of gas heating, W·sec^{-1}; R_{in}, coefficient of the force of ion–neutral friction, sec^{-1}; **r**, radius-vector of the point on the trajectory; S_{ij}, coefficient of the force of ion–ion friction, sec^{-1}; s, length of the force line arc, m; T, temperature of particles, K; t, time, sec; V, velocity of particles, m·sec^{-1}; **v**, group velocity vector; Y^+, sum concentration of positive ions-links, m^{-3}; Y^-, sum concentration of negative ions-links, m^{-3}; Z, distance to the point of receipt, m; z, height, m; α, β, beam coordinates of a source; α_i, coefficient of recombination of particles, sec^{-1}; $\Gamma = 1/2[\mathbf{p}^2 - n^2(\mathbf{r})]$, Hamiltonian; γ, coefficient of photoungluing; ζ, angle between the wave vector and group velocity, deg; η, coefficient of polarization; θ, latitude, deg; λ_{ie}, thermal conductivity of ions, electrons, W·m^{-2}·K^{-1}; ν, frequency of collisions, sec^{-1}; ξ, coefficient of kinematic viscosity, m^2·sec^{-1}; τ, time, sec; φ, width, deg; χ, coefficient of absorption; Ψ, phase of a wave, deg; ω, angular frequency, deg. Subscripts: e, electron; i, j = 1, 2, ..., 6, numbers of ions; k, n = 1, 2, ..., 19, numbers of neutral components; ef, effective; m, molecular; n, neutral; t, turbulent.

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